The Dividend Discount Model
Retaining and Reinvesting Free Cash Flow

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The dividend discount model (DDM) defines company value as the discounted value of expected future free cash flow. Even though the dividend discount model assumes that free cash flow is paid out as dividends more often than not the dividend yield parameter in the option pricing model is set to zero, which means that free cash flow is being retained by the company and reinvested at the discount rate in the real world and at the risk-free rate in the risk-neutral world.

In this white paper we will build a model for the dividend cash account where additions to the cash account are dividends retained plus interest earned on those retained dividends. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are tasked with building a model to forecast ABC Company future stock price given the following go-forward model assumptions...

Table 1: Go-Forward Model Assumptions

Description	Value
Annualized free cash flow at time zero ($)	1,000,000
Cash flow growth rate (%)	5.00
Risk-free interest rate (%)	3.00
Risk-adjusted discount rate (%)	21.00

Our task is to answer the following questions assuming that monthly cash flows are discounted...

Question 1: What is expected stock price at the end of year 5 given that the dividend yield is zero?

Question 2: What is expected stock price at the end of year 5 given the dividend yield implied by the DDM?

Question 3: What is expected stock price at the end of year 5 given that free cash flow is deposited into a company-owned cash account rather than being distributed as dividends to owners?

DDM Implied Equation For Future Stock Price

We will define the variable N to be the number of periods in one year and the variable C_m to be periodic caslh flow at the end of month m. The equation for periodic cash flow at time zero is...

$$C_0 = \text{Annualized cash flow} \times \frac{1}{N} \tag{1}$$

We will define the variable g to be the periodic cash flow growth rate. The equation for the periodic growth rate is...

$$g = \left(1 + \text{annual cash flow growth rate}\right)^{\frac{1}{N}} - 1 \tag{2}$$

We will define the variable k to be the periodic discount rate. The equation for the periodic discount rate is...

$$k = \left(1 + \text{annual discount rate}\right)^{\frac{1}{N}} - 1 \tag{3}$$

We will define the variable V_m to be company value at the end of period m. Using Equations (1), (2) and (3) above the equation for company value at time zero is...

$$V_0 = \sum_{n=1}^{\infty} C_0 \left(1+g\right)^n \left(1+k\right)^{-n} \tag{4}$$

Using Equation (4) above the equation for company value at the end of period m is...

$$V_m = \sum_{n=1}^{\infty} C_0 \left(1+g\right)^{m+n} \left(1+k\right)^{-n} = C_0 \left(1+m\right)^m \sum_{n=1}^{\infty} \left(1+g\right)^n \left(1+k\right)^{-n}$$

Note that we can rewrite Equation (5) above as...

$$V_m = C_0 \left(1+g\right)^m \sum_{n=1}^{\infty} \theta^n \text{ ...where... } \theta = \frac{1+g}{1+k} \tag{5}$$

The solution to the summation in Equation (5) above is...

$$\text{if... } \sum_{n=1}^{m} \theta^n = \frac{\theta - \theta^{m+1}}{1-\theta} \text{ ...then... } \sum_{n=1}^{\infty} \theta^n = \lim_{m \to \infty} \frac{\theta - \theta^{m+1}}{1-\theta} = \frac{\theta}{1-\theta} = \frac{1+g}{k-g} \text{ ...when... } 0 < \theta < 1 \tag{6}$$

Using Equation (6) above the solution to Equation (5) above is...

$$V_m = C_0 \left(1+g\right)^m \left(\frac{1+g}{k-g}\right) = C_0 \left(1+g\right)^{m+1} \left(k-g\right)^{-1} \tag{7}$$

Company Value as a Function of the Cash Flow Growth Rate

We will define the variable ΔV_m to be the percentage change in company value over time interval $[m, m+1]$. The equation for the percentage change in company value is...

$$\Delta V_m = \left(V_{m+1} - V_m\right) \Big/ V_m \tag{8}$$

Using Equation (7) above we can rewrite Equation (8) above as...

$$\Delta V_m = \left(C_0\,(1+g)^{m+2}\,\Gamma - C_0\,(1+g)^{m+1}\,\Gamma\right) \Big/ C_0\,(1+g)^{m+1}\,\Gamma \text{ ...where... } \Gamma = \left(k-g\right)^{-1} \tag{9}$$

The solution to Equation (9) above is...

$$\Delta V_m = \frac{C_0\,(1+g)^{m+1}}{C_0\,(1+g)^{m+1}} \left((1+g) - 1\right) = g \tag{10}$$

Using Equation (10) above the equation for company value at the end of period m as a function of the cash flow growth rate is...

$$V_m = V_0 \left(1+g\right)^m \tag{11}$$

Value as a Function of Discount Rate and Dividend Yield

If we discount at rate k then we should earn at the same rate. The following equation defines the percentage change in company value over the time interval $[m, m+1]$ to be a function of company value at the beginning of the time interval, the discount rate and the dividend yield...

$$\Delta V_m = \left(k\,V_m - d\,V_m\right) \Big/ V_m = k - d \tag{12}$$

Using Equation (12) above the equation for company value at the end of period m as a function of the discount rate and dividend yield is...

$$V_m = V_0 \left(1 + k - d\right)^m \tag{13}$$

If we equate Equations (11) and (13) above then we can make the following statement...

$$\text{if...} \;\; V_0 \left(1 + g\right)^m = V_0 \left(1 + k - d\right)^m \;\; \text{...then...} \;\; g = k - d \;\; \text{...such that...} \;\; d = k - g \tag{14}$$

The Dividend Cash Account

We will define the variable B_m to be the dividend cash account balance at the end of period m. Rather than pay out dividends to owners free cash flow is deposited into a company-owned cash account earning an interest rate of r. The equation for cash account balance at the end of period m is...

$$B_m = \sum_{n=1}^{m} d\, V_{n-1} \left(1 + r\right)^{m-n} \tag{15}$$

We defined the variable r to be the cash account periodic rate of interest. The equation for the periodic interest rate is...

$$r = \left(1 + \text{annual interest rate}\right)^{\frac{1}{N}} - 1 \tag{16}$$

Using Equation (11) above we can rewrite Equation (15) above as...

$$B_m = \sum_{n=1}^{m} d\, V_0 \left(1 + g\right)^{n-1} \left(1 + r\right)^{m-n} \tag{17}$$

We can rewrite Equation (17) above as...

$$B_m = d\, V_0 \left(1 + r\right)^m \left(1 + g\right)^{-1} \sum_{n=1}^{m} \Pi^n \;\; \text{...where...} \;\; \Pi = \frac{1 + g}{1 + r} \tag{18}$$

The equation for the solution to the summation in Equation (18) above is...

$$\sum_{n=1}^{m} \Pi^n = \frac{\Pi - \Pi^{m+1}}{1 - \Pi} \tag{19}$$

Using Equation (19) above the solution to Equation (18) above is...

$$B_m = d\, V_0 \left(1 + r\right)^m \left(1 + g\right)^{-1} \left(\frac{\Pi - \Pi^{m+1}}{1 - \Pi}\right) \;\; \text{...where...} \;\; \Pi = \frac{1 + g}{1 + r} \tag{20}$$

Note that Equation (20) above assumes that all dividends are retained by the company and reinvested. If the company wishes to retain a percentage of free cash flow such that a portion of dividends are deposited into a company-owned cash account with the remainder paid out as dividends then Equation (20) becomes...

$$B_m = d\, \Gamma\, V_0 \left(1 + r\right)^m \left(1 + g\right)^{-1} \left(\frac{\Pi - \Pi^{m+1}}{1 - \Pi}\right) \;\; \text{...where...} \;\; \Gamma = \text{Percent of free cash flow retained} \tag{21}$$

Answers To Our Hypothetical Problem

Using Equation (1) above and the data in Table 1 above the equation for the periodic cash flow at time zero is...

$$C_0 = 1{,}000{,}000 \times \frac{1}{12} = 83{,}333 \tag{22}$$

Using Equation (2) above and the data in Table 1 above the equation for the periodic cash flow growth rate is...

$$g = \left(1 + 0.05\right)^{\frac{1}{12}} - 1 = 0.00407 \tag{23}$$

Using Equation (3) above and the data in Table 1 above the equation for the periodic discount rate is...

$$k = \left(1 + 0.21\right)^{\frac{1}{12}} - 1 = 0.01601 \tag{24}$$

Using Equation (16) above and the data in Table 1 above the equation for the periodic risk-free rate is...

$$r = \left(1 + 0.03\right)^{\frac{1}{12}} - 1 = 0.00247 \tag{25}$$

Using Equation (7) above and the parameter estimates above the equation for company value at time zero is...

$$V_0 = 83,333 \times \left(1 + 0.00407\right)^{1} \times \left(0.01601 - 0.00407\right)^{-1} = 7,009,000 \tag{26}$$

Question 1: What is expected stock price at the end of year 5 given that dividend yield is zero?

Using Equation (13) above and the parameter estimates above the answer to the question is...

$$V_{60} = 7,009,000 \times \left(1 + 0.01601 - 0\right)^{60} = 18,179,800 \tag{27}$$

Question 2: What is expected stock price at the end of year 5 given dividend yield implied by the DDM?

Using Equations (13) and (14) above and the parameter estimates above the answer to the question is...

$$V_{60} = 7,009,000 \times \left(1 + 0.01601 - 0.01194\right)^{60} = 8,945,600 \text{ ...where... } d = 0.01601 - 0.00407 = 0.01194 \tag{28}$$

Question 3: What is expected stock price at the end of year 5 given that free cash flow is deposited into a company-owned cash account rather than being distributed as dividends to owners?

Using Equation (20) above and the parameter estimates above the equation for the multiplier Π is...

$$\Pi = \frac{1 + 0.00407}{1 + 0.00247} = 1.00160 \tag{29}$$

Using Equations (20) and (29) above and the parameter estimates above the equation for the cash account balance is...

$$B_{60} = 0.01194 \times 7,009,000 \times \left(1 + 0.00247\right)^{60} \left(1 + 0.00407\right)^{-1} \times 1.00160 = 6,089,100 \tag{30}$$

Using Equations (28) and (30) above the answer to the question is...

$$V_{60} = 8,945,600 + 6,089,100 = 15,034,700 \tag{31}$$